

Numerical Study of Natural Convection Heat Transfer in Enclosures with Conducting Fins Attached to a Vertical Sidewall

Ahmed F. Alfahaid* and R.Y. Sakr

Mechanical Technology Department, Riyadh College of Technology

P.O. Box 91471 Riyadh 11633, Saudi Arabia

*dralfahaid@yahoo.com

Abstract:

Laminar natural convection in vertical enclosures with conducting fins attached to the hot vertical wall has been studied numerically. Side vertical walls were kept at constant but different temperatures, while the horizontal top and bottom walls were kept adiabatically. A conjugate formulation was used for the mathematical formulation of the problem and a computer program based on the control volume approach and the SIMPLEC algorithm was used. The main aim of the current paper is to study numerically the effects of the number of fins and fins length as well as Rayleigh number on the flow pattern and heat transfer. It was observed that the heat transfer rate through an enclosure is affected greatly and hence can be controlled by the number of the attached fins to the vertical side wall of the enclosure as well as the fin lengths.

1. Introduction

Due to its low cost and apparent simplicity, natural convection cooling or heating has always been an attractive technique in thermal engineering. The applications in which natural convection cooling or heating are countless, embracing many technical fields. Examples are energy storage systems, solar applications, material processing, nuclear reactors, heat exchangers, thermal control of electronic devices and buildings. Such variety of applications has generated large body of work that studied buoyancy driven flows in several configurations and different boundary conditions.

Nowadays, the increasing demand for greater compactness is accompanied by a corresponding increase in surface power dissipation. There is therefore a current call to pursue better configurations in order to maximize heat transfer rate. So, studies on natural convection with active sources of imposed temperature are important from theoretical and practical points of view. The relative positions and sizes of the active sources determine the flow patterns and heat transfer rate. Very few studies dealt with optimization of thermal performance [1]. Most of the previous studies found are of descriptive nature [2, 3]. Zimmerman and Acharaya [4] studied the natural convection heat transfer in enclosures with baffles attached to the

conducting top and bottom walls. They found that the average Nusselt number for enclosure with baffles is significantly less than the unbaffled enclosure.

Frederick [5] studied the natural convection in square cavities with a diathermal divider attached to the cold wall. Significant heat transfer reductions relative to the undivided cavity were found for $10^3 \leq Ra \leq 10^5$. The same problem with a conducting divider with varying fin length and thermal conductivity is solved by Frederick and Valenica [6]. With low thermal conductivity, divider reduction of heat transfer was significant at $10^3 \leq Ra \leq 10^5$. An increase in heat transfer was obtained at lower Rayleigh numbers with partitions of high thermal conductivities. Buoyancy driven convection in a rectangular enclosure fitted with a vertical adiabatic partition is investigated experimentally for different configurations of the partition by Khalifa and Sahib [7]. They reported correlations and percentage reduction in heat transfer for each case as compared to that without partition. Natural convection in cubical enclosures with square thermal sources of different sizes on adjacent vertical wall and imposed temperatures was investigated numerically by Fredrick and Berbakov [8]. They found that the flow patterns are completely asymmetric depending on Rayleigh number and dimensions of the hot source.

The rate of heat transfer through a fluid layer in an enclosure may be controlled by attaching fins to one of the active walls of the enclosure; the heat transfer rate may be reduced or enhanced. Hence, studying heat transfer through a fluid layer enclosed in a cavity with fins attached to its wall has value from practical as well as theoretical point of views. Fins are also used in flow channels to control the heat transfer rate across the channel walls. In the literature, there exist studies on the effects of fins of the fluid flow and heat transfer characteristics in channels with fins attached to the channel walls [9-13].

Low-level turbulence natural convection in an air filled vertical partitioned square cavity was studied experimentally by Ampofo [14]. His results showed that the used partitions length tends to reduce heat transfer rate along with the hot wall compared with similar cavities without partitions. Also, the computational modeling, the temperature

Nomenclatures

A: Enclosure aspect ration, H/W
H: Enclosure height
h: Micro-cavity height
k: Thermal conductivity
L: Fin length
Nu: Nusselt number
n: Direction normal to the surface
p: Pressure
Pr: Prandtl number, ν_f/α_f
 q_h : Heat flux on hot wall and fins
 q_c : Heat flux on cold wall
Ra: Rayleigh number, $g \beta (T_h - T_c) W^3 / \nu_f \alpha_f$
T: Temperature
t: Fin thickness
U: Dimensionless velocity component in X-direction
 \vec{V} : Velocity vector
V: Dimensionless velocity component in Y-direction

W: Enclosure width
x, y : Cartesian coordinates
X, Y: Dimensionless Cartesian coordinates

Greek

α : Thermal diffusivity
 β : Thermal expansion coefficient
 ν : kinematic viscosity
 θ : Dimensionless temperature $(T - T_m) / (T_h - T_c)$

Subscripts

c: cold
f: fluid
h: hot
m: mean value
r: relative
s: solid

measurement and flow visualization were conducted to investigate the enhancement of natural convection, conduction and radiation heat transfer from a discrete heat source in an enclosure using pin fin heat sink was investigated by Yu and Joshi [15]. The pin fin was attached to the heated component, which was flush mounted on the enclosure base. The effect of fin aspect ratio and length of a high conductivity of a single rectangular fin attached to the hot wall of a three-dimensional differentially heated enclosure in laminar natural convection is studied numerically by Silva and Gosselin [16]. The numerical results showed that for an enclosure assisted with a large volume fraction fin, the fin aspect ratio does not play an important role and the average heat flux transferred to the fluid increases monotonically with the horizontal length. Yucel and Turkoglu, [17] studied numerically laminar natural convection in vertical enclosures with conducting fins attached to the cold vertical wall. They studied the effect of fin configuration and Rayleigh number on the flow structure and heat transfer. They observed that the heat transfer rate through an enclosure can be controlled by attaching fins to the walls of the enclosure.

The main aim of the present paper is to investigate numerically the nature of buoyancy driven flow inside finned enclosure and the heat transfer characteristics on both hot finned and cold sidewalls.

2. Physical and mathematical formulation

The geometry of the present problem is shown in Fig. (1). The height and width of the enclosure are denoted by H, W respectively. The thickness and the length of the fins are denoted by t and L respectively. The depth of the enclosure is assumed to be long. Hence, the problem can be considered two-dimensional. Using the Bousinesq approximation to account for the density variation in buoyancy term of the momentum equation and neglecting the

dissipation effect due to viscous term the governing equations in both the fluid (air) and solid (fins) phases in non-dimensional form are written as follows:

$$\nabla \cdot \vec{V} = 0 \quad (1)$$

$$(\nabla \cdot \vec{V}) \vec{V} = -\nabla P + \text{Pr} \nabla^2 \vec{V} + \text{Ra} \text{Pr} \theta \underline{j} \quad (2)$$

$$\vec{V} \cdot \nabla \theta = \nabla^2 \theta \quad (3)$$

$$k_r \nabla^2 \theta = 0 \quad (4)$$

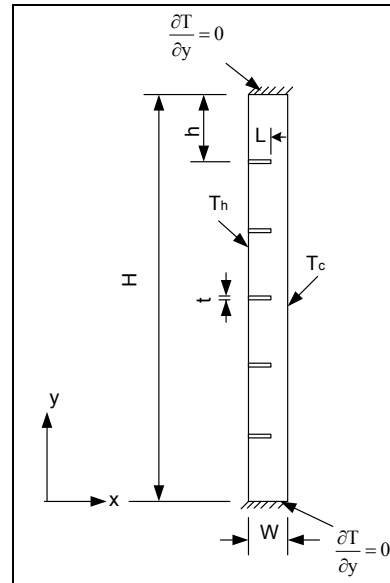


Fig. (1) The present physical model

The non-dimensional mathematical representations of the boundary conditions are:

$$\theta = 1, U = V = 0 \text{ at } X = 0$$

$$\theta = U = V = 0 \text{ at } X = 1$$

$$\frac{\partial \theta}{\partial Y} = U = V = 0 \text{ at } Y = 0, Y = A$$

At the solid-fluid interfaces, the following compatibility conditions apply:-

$$U = V = 0, \theta_f = \theta_s, \left(\frac{\partial \theta}{\partial n}\right)_f = k_r \left(\frac{\partial \theta}{\partial n}\right)_s$$

Equations (1-4) were solved together with the relevant boundary conditions to determine the velocity and temperature fields in the enclosure. The stream function was calculated by using the velocity field. The average Nusselt number at the hot wall and fins is given by:-

$$Nu_h = \frac{q_h}{(T_h - T_c)} \frac{W}{k} = \int_0^{A+2N(L/W)} \frac{\partial \theta}{\partial n} dn$$

Also, the average Nusselt number at the cold wall is given by:-

$$Nu_c = \frac{q_c}{(T_h - T_c)} \frac{W}{k} = \int_0^A \frac{d\theta}{dX} \Big|_{X=1} dY$$

3. Numerical Procedure

In this study, CFD software FLUENT [18] was utilized to simulate the natural convection case in cavity for different fins configurations. The numerical model is based on finite volume method as discussed by Patankar [19]. Regarding the boundary condition, no-slip condition was used along the walls and fins, moreover constant and different temperatures were applied on both left and right vertical walls while adiabatic condition was used on the top and bottom walls. The present code utilizes the segregated method which employed SIMPLEC algorithm with second upwind scheme for the integration of both the momentum and energy equations.

The computations were performed on an average of 8000 unstructured grid for all cases. The convergence criterion adopted in the calculations was set so that the normalized residual was less than 10^{-6} at any grid node.

4. Validation and Verification

To check the consistency and reliability of the present analysis, its results compared with the available published data. The comparison was carried out for $A=10$, $t/W=0.1$, $Pr=0.71$, $k_r=30$ and $L/W=0.25$. A good agreement between the present code and the available data in literature [17] is illustrated as shown in Fig. (2).

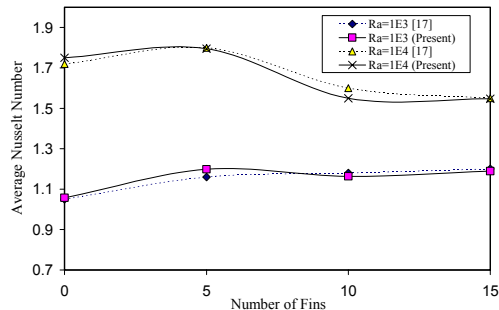


Fig. (2) Comparison of the average Nusselt number between the present prediction and literature

5. Results and Discussions

Rayleigh number and geometric parameters such as cavity aspect ratio, fin length, and number of fins govern the heat transfer and flow characteristics. In the present work, a cavity of aspect ratio $A=10$ with fins attached to its hot wall was considered. The fins are made of aluminum and their dimensionless thickness $t/W=0.1$ is kept constant. The enclosure contains air, and hence Pr was assumed 0.71. Computations were performed at different Rayleigh numbers based on the enclosure width in the range of $Ra = 10^3$ up to 10^5 , for dimensionless fin length (L/W) of 0.2, 0.4 and 0.6 and the number of fins of 0, 5, 10 and 15. To observe the effects of these parameters on the fluid flow and the heat transfer mechanism in the cavity, the streamline and isotherm contours in the cavity were plotted and the average Nusselt number on both hot and cold sides are calculated.

Figure (3) shows the streamline contours for different fin lengths ($L/W=0.2, 0.4$, and 0.6) and different number of fins (0, 5, 10 and 15) at $Ra=10^4$. The streamline contours in Fig. (3) show how the number of fins affects the flow pattern in the cavity. In this figure, there is only one circulating cell in the unfinned cavity, the fluid flows upward near the hot wall and downward near the cold wall, turning at the insulated end walls. When fins are attached to the wall of the cavity the flow field changes to a multi-cellular circulating pattern with a primary circulation outside. The structure of this multi-cellular flow pattern changes with varying the number of fins. With further increase in the number of fins, again a single cell flow pattern forms when the number of fins $N \geq 15$ which leads to separation of the fluid layer over the hot wall and consequently decrease the average Nusselt number on the hot wall while on the cold wall the situation is different.

Figure (4) shows the effect of the number of fins and fin length on the isotherm contours at $Ra=10^4$. The figure shows that the isotherms have larger flat portion in case of $N=5$, $L/W=0.2, 0.4$ and 0.6 which indicates that the natural convection is the dominant mode of heat transfer. As the number of fins increases ($N \geq 10$) the isotherms become more parallel to either the right cold wall or to the curve connecting the successive fins tips and the midpoint of the micro-cavity on the left hot wall which refers that conduction becomes a dominant mode of heat transfer.

The effect of Rayleigh number on the flow characteristics is represented as stream function contours for $N=10$ and $L/W=0.4$ in Fig. (5). The flow patterns for $Ra=10^3$ and 10^4 are nearly similar while the flow pattern for $Ra=10^5$ is different. The corresponding isothermal lines contours are illustrated in Fig. (6). Also, it is noted that, the

average Nusselt number at both hot and cold walls increases as Rayleigh number increase, Figs. (7-8). Figure (7) shows the variation of the average Nusselt number at the hot wall (including the attached fins) with the number of fins for different fin length at different Rayleigh numbers. It is noticed that for all Rayleigh number $Ra=10^3-10^5$, the average Nusselt number increases with increasing the number of fins up to certain number of fins (depending on Rayleigh number) and then decreases with further increase of the number of fins. Also, the figure illustrates that for $Ra=10^5$, the effect of fin length on the average Nusselt number is more significant and the average Nusselt number decreases with the increase of fin length. This may be attributed to the boundary layer separation. The variation of the average Nusselt number at the cold wall with the number of fins for different fin length at different Rayleigh numbers is illustrated in Fig. (8). From this figure, it is shown that at low Rayleigh number $Ra=10^3$, as the fin number increases, the average Nusselt number increases for all fin lengths. Also, the average Nusselt number increases with the increase of fin length for $Ra \geq 10^4$ and fin length $L/W=0.2$ the average Nusselt number decreases with the increase of number of fins. For $Ra \geq 10^4$ and fin length 0.4 and 0.6, the average Nusselt number increases with the increase of the number of fins up to certain fin number depending on Rayleigh number, the further increase of numbers of fins leads to decrease in the average Nusselt number. So, it can be concluded that for fin length 0.2, no heat transfer enhancement is noticed at the cold surface for all values of Rayleigh number.

6. Conclusions

From the findings in current paper, it can be concluded that:

1. For all Rayleigh numbers, an increase in the average Nusselt number on the hot wall and fins is noticed with the increase of the number of fins compared with the unfinned cavity up to certain number of fins, then increasing the number of fins leads to decrease in the average Nusselt number.
2. For low value of Rayleigh numbers, $Ra=10^3$, and $L/W=0.6$ the average Nusselt number at the hot side wall and fins has the lowest value for any number of attached fins. However, for $L/W=0.2$ and 0.4 the larger the fins length at $N > 10$ the lower value of the average Nusselt number. For $Ra=10^4$ and $N < 7$ the greater the fins length results in the higher Nusselt number and the situation is reversed for $N > 7$. For high value of Ra ($Ra=10^5$), as the fins length increases the average Nusselt number decreases.
3. For $Ra=10^3$ and fin length $L/W=0.2$, insignificant enhancement of the average Nusselt number on the cold sidewall is observed. While for fin length $L/W=0.4$ and 0.6, the

average Nusselt number increases with the increase of the number of fins.

4. For $Ra \geq 10^4$ and $L/W=0.2$, no enhancement of the average Nusselt number on the cold sidewall is observed with increase of the number of fins.
5. For $Ra \geq 10^4$ and $L/W=0.4$ and 0.6, an increase of the average Nusselt number on the cold sidewall is observed with increase of the number of fins up to a certain number, then further increase of the number of fins leads to decrease of the average Nusselt number.
6. As fin length increases the average Nusselt number on the cold sidewall increases for all Rayleigh numbers.
7. As Rayleigh number increases the average Nusselt number on both hot and cold and sidewalls increases for all Rayleigh numbers.

References:

1. A.K. da Silva; S. Lorente, A. Bejan, "Optimal distribution of discrete heat sources on a wall with natural convection", *Int. J. Heat Mass Transfer* 47 (2004) 203-214.
2. G.N. Facas, "Natural convection in a cavity with fins attached to both vertical walls", *J. Thermophys Heat Transfer* 32 (1997) 365-373.
3. E.K. Lakhali, M. Hasnaoui, E. Bilgen, P. Vasseur, "Natural convection in inclined rectangular enclosures with perfectly conducting fins attached on the heated walls", *J. Heat Mass Transfer* 32, (1997) 365-373.
4. E. Zimmerman, S. Acharya, "Free convection heat transfer in a partially divided vertical enclosure with conducting end wall", *Int. J. Heat Mass Transfer* 30 (1987) 319-331.
5. R.L. Frederick, "Natural convection in an inclined square enclosure with a partition attached to its cold wall", *Int. J. Heat Mass Transfer* 32 (1989) 87-94.
6. R.L. Frederick, A. Valencia, "Heat Transfer in a square cavity with conducting partition on its hot wall", *Int. Comm. Heat Transfer* 16 (1989) 347-354.
7. A.J.N. Khalifa, W.K. Sahib, "Turbulent buoyancy driven convection in partially divided enclosures", *Energy Conservation and Management*, 43 (2002) 2115-2121.
8. R.L. Frederick, O. Berbakov, "Natural convection in cubical enclosures with thermal sources on adjacent vertical walls", *Num. Heat Transfer, part A*, 41 (2002), 331-340.
9. J.J. Hawang, T.M. Liou, "Heat transfer and friction in a low aspect ratio rectangular channel with staggered perforated ribs on two opposite walls", *J. Heat Trans.* 117 (1995) 843-850.
10. C.H. Cheng, I.J. Yang, "Buoyancy induced recirculation bubbles and heat convection of developing flow in vertical channel with fin arrays", *Int. J. Heat and Fluid Flow* 15 (1994) 11-19.

11. T.M. Liou, W.B. Wang, Y.J. Chang, "Holographic interferometry study of spatically periodic heat transfer in a channel with ribs detailed from one wall", *J. Heat Transfer* 117 (1995) 32-39.
12. J. Devalath, Y. Bayazitoglu, "Forced convection cooling across rectangular blocks", *J. Heat Transfer* 109 (1987) 321-328.
13. Y.M. Zhang, W.Z. Gu, J.C. Han, "Heat transfer and friction in rectangular channel with ribbed or ribbed-grooved walls", *J. Heat Transfer* 116 (1994) 58-65.
14. F. Ampofo, "Turbulent natural convection in an air filled partitioned square cavity", *Int. J. Heat and Fluid Flow* 25 (2004) 103-114.
15. E. Yu, Y. Joshi, "Heat transfer enhancement from enclosed discrete components using pin-fin heat sinks", *Int. J. Heat Mass Transfer* 45 (2002) 4957-4966.
16. A.K. da Silva, L. Gosselin, "On the thermal performance of an internally finned three-dimensional cubic enclosure in natural convection", *Int. J. Thermal Sciences*, in press.
17. N. Yucel, H. Turkoglu, "Numerical analysis of laminar natural convection in enclosures with fins attached to an active wall", *Heat and Mass Transfer* 33 (1998) 307-314 - Springer-Verlag.
18. Fluent Version 6 User's Guide, Fluent Incorporated, New Hampsier, 2003.
19. S.V. Patankar, in: *Numerical Heat Transfer and Fluid Flow*, Mc-Graw Hill, New York, 1980.

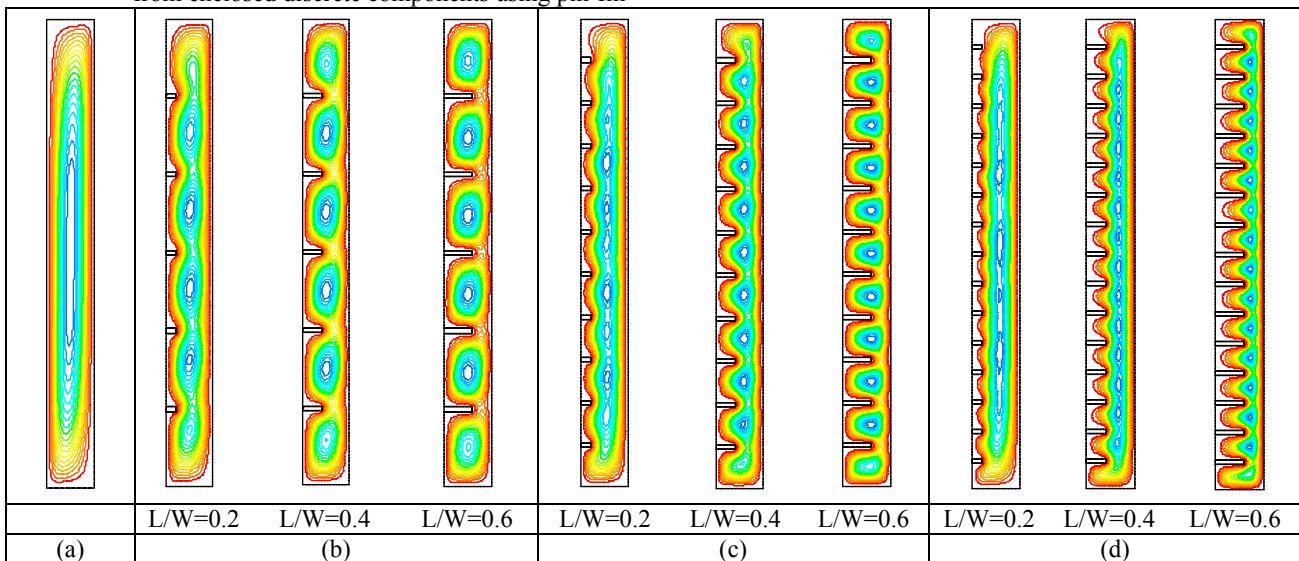


Fig. (3) Stream function contours at $Ra=10^4$ for different fin length and number of fins a-N=0, b-N=5, c-N=10 and d-N=15

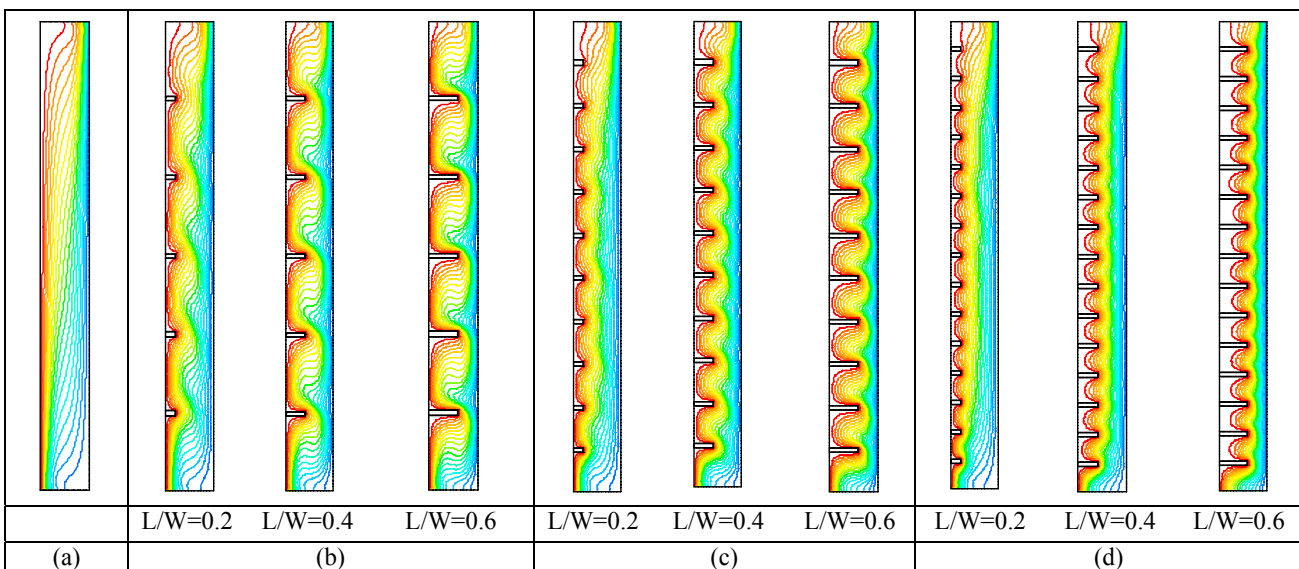


Fig. (4) Isothermal lines contours at $Ra=10^4$ for different fin length and number of fins a-N=0, b-N=5, c-N=10 and d-N=15

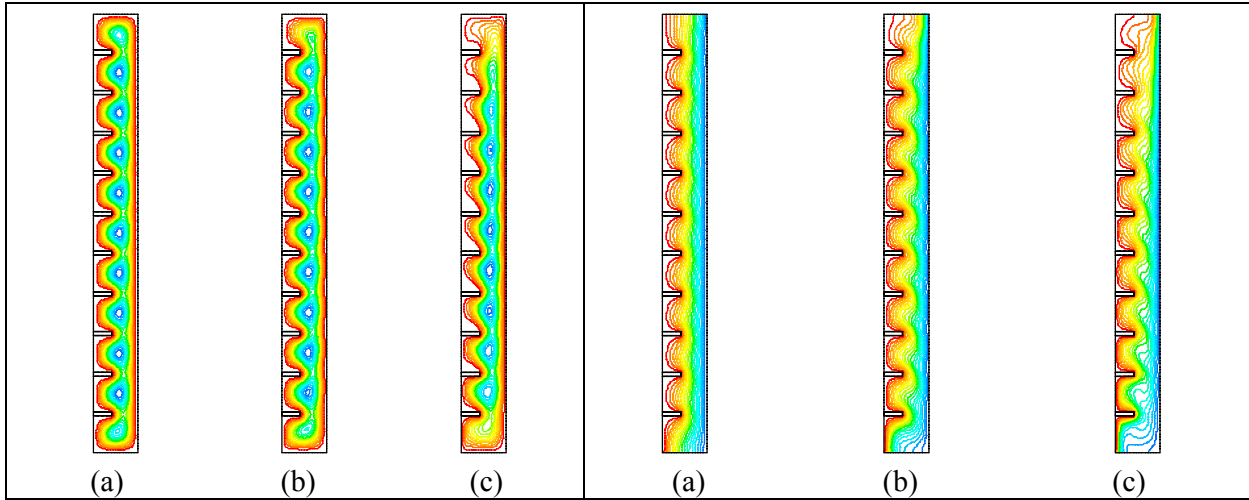


Fig. (5) Stream lines contours for $N=10$, $L/W=0.4$ at different Rayleigh number a- $Ra=10^3$, b- $Ra=10^4$, c- $Ra=10^5$

Fig. (6) Stream lines contours for $N=10$, $L/W=0.4$ at different Rayleigh number a- $Ra=10^3$, b- $Ra=10^4$, c- $Ra=10^5$

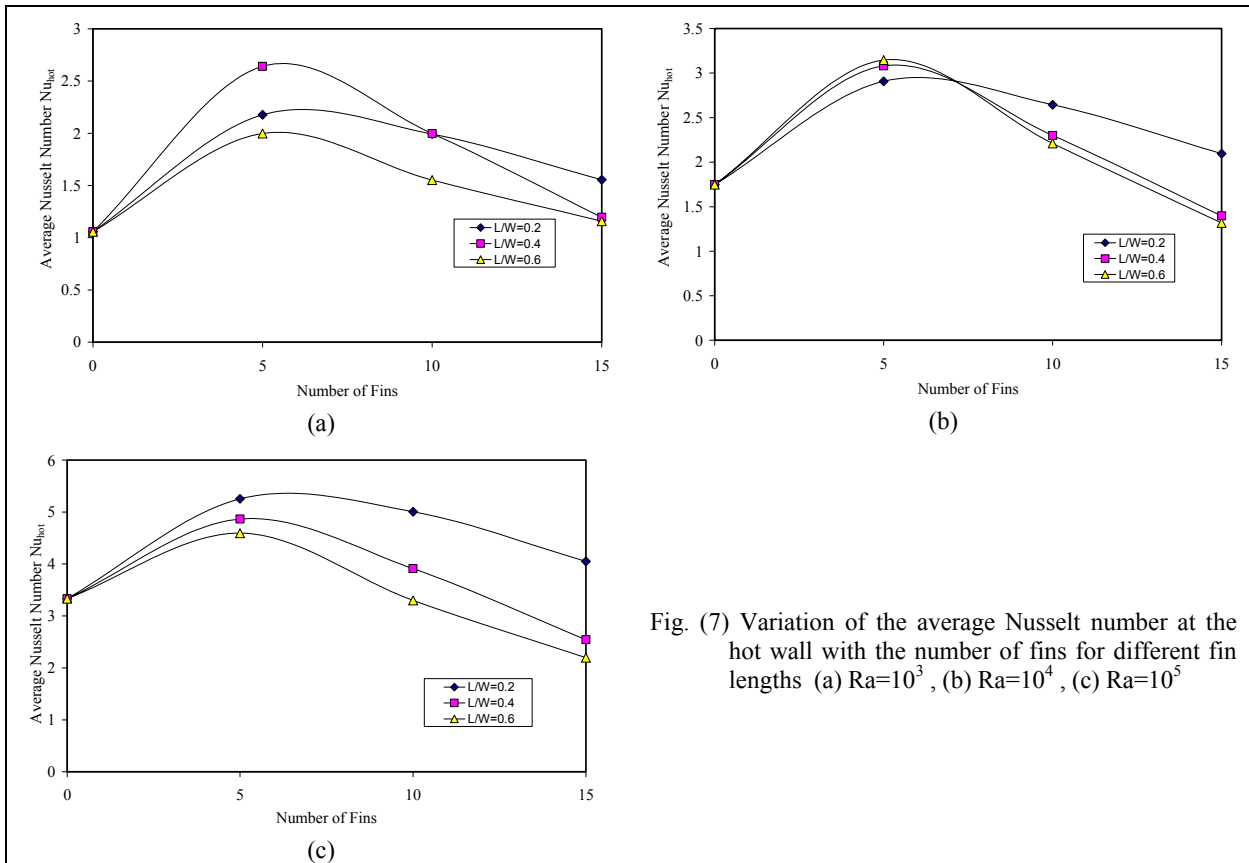


Fig. (7) Variation of the average Nusselt number at the hot wall with the number of fins for different fin lengths (a) $Ra=10^3$, (b) $Ra=10^4$, (c) $Ra=10^5$

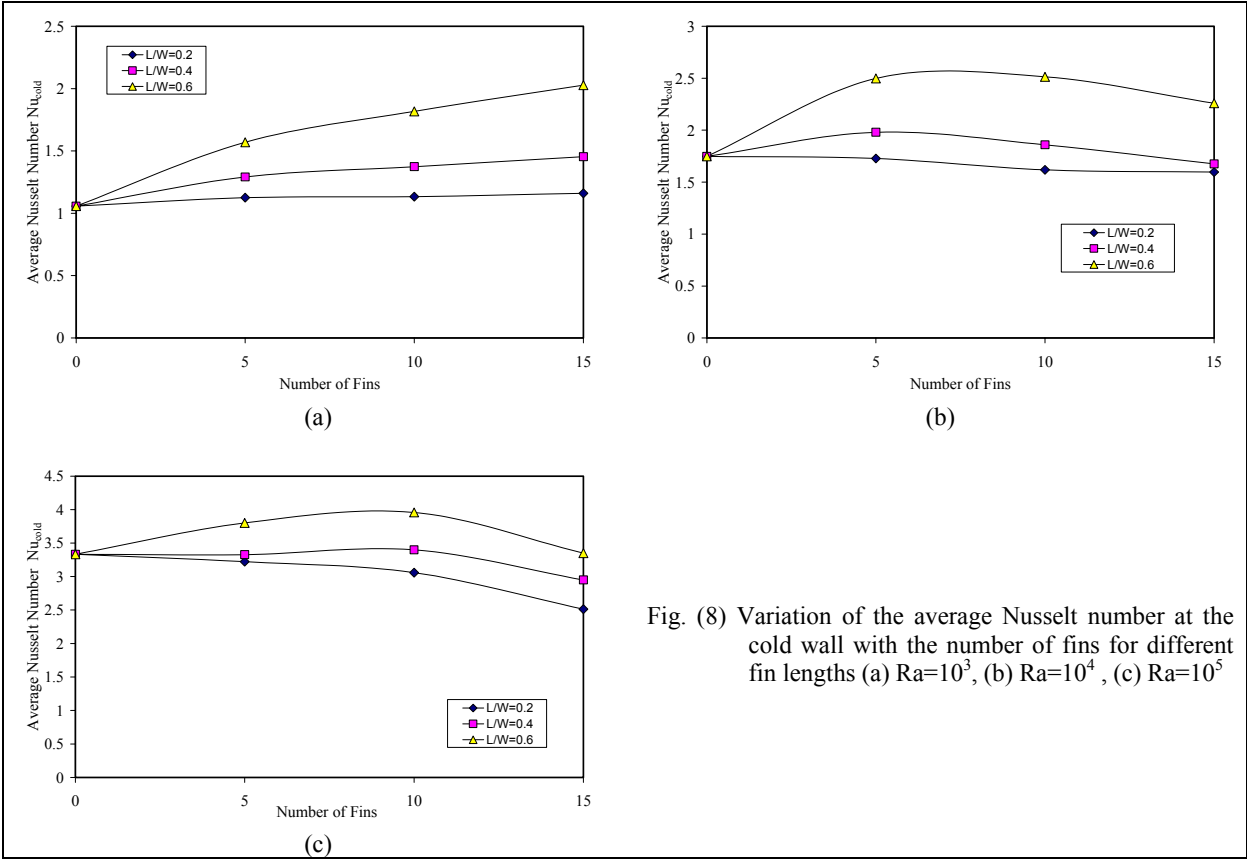


Fig. (8) Variation of the average Nusselt number at the cold wall with the number of fins for different fin lengths (a) $Ra=10^3$, (b) $Ra=10^4$, (c) $Ra=10^5$